A credit risk model with a switching barrier and asymmetric information

Ralf Wunderlich

Brandenburg University of Technology Cottbus - Senftenberg

Joint work with	*Imke Redeker*	*(Cottbus)*
	Michaela Szölgyenyi	*(WU Wien/ETH Zürich)*

Workshop Mathematik in Forschung und Lehre, Waschleithe, September 12–14, 2017

Mathematical modeling of credit risk

Structural approach:

- Default is triggered when the firm's assets fall to some threshold
- Default barrier can be imposed exogenously or endogenously
- Bond investors know the model inputs and parameters
- Problem: no short-term default risk

Reduced-form approach:

- Default event is exogenously given
- Stochastic structure of default is characterized by an intensity (conditional default rate)
- Short spread is directly given by the default rate
- Problem: economic interpretation

Links between structural and reduced-form approaches

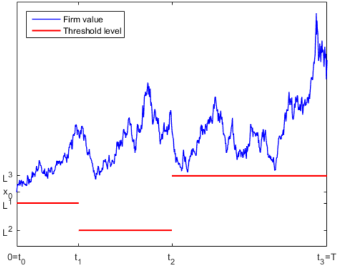
Methods to introduce short-term default risk in a structural model:

- Default barrier is a random variable, e.g.
  Lando (1998) , Giesecke & Goldberg (2008),
  Hillairet & Jiao (2012)

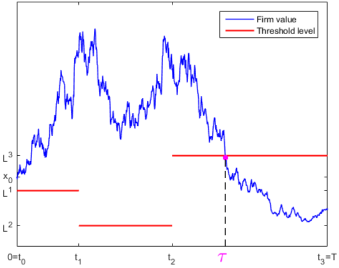
- Firm's assets process is partially observed by investors, e.g.
  Duffie & Lando (2001), Lakner & Liang (2008),
  Jeanblanc & Valchev (2005)

Model framework

No Default



Default

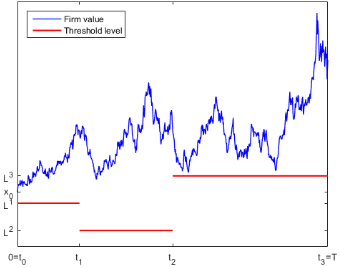


Notation	X	total value of the firm's assets
	L	threshold level which triggers default
	τ	(random) time of default
	T	finite time horizon

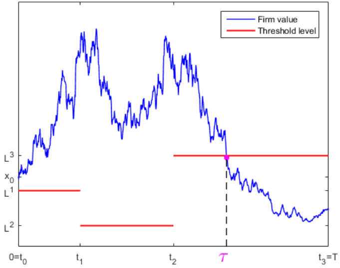
Default occurs when the firm's assets X fall
to the default threshold (barrier) L for the first time.

Model framework

No Default



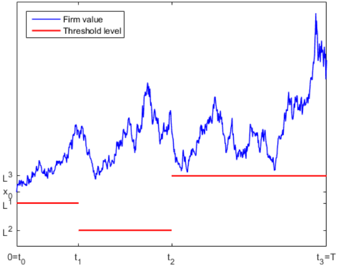
Default



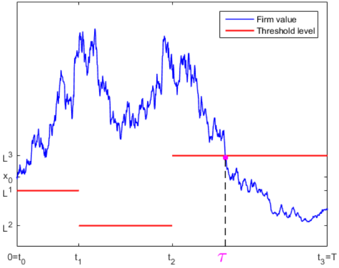
Probability space	$(\Omega, \mathcal{A}, \mathbb{P})$
Filtration	$\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$ generated by X
Asset process	$\mathrm{d}X_t = X_t(\mu\,\mathrm{d}t + \sigma\,\mathrm{d}B_t),\ X_0 = x_0$
	where $(B_t)_{t\in[0,T]}$ is a $\mathbb{F}$-Brownian motion,
	$\mu \in \mathbb{R}$ and $\sigma > 0$. We denote $m := \mu - \sigma^2/2$.

Model framework

No Default



Default



Default barrier $L_t = \sum_{i=1}^{n} L^i \mathbf{1}_{[t_{i-1}, t_i)}$

where L^i is a $\mathcal{A}$-measurable random variable

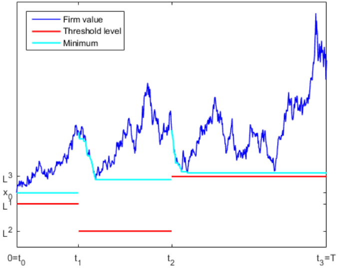
and t_i, $i = 0, \ldots, n$, are deterministic time points

such that $0 = t_0 < t_1 < \ldots t_{n-1} < t_n = T$.

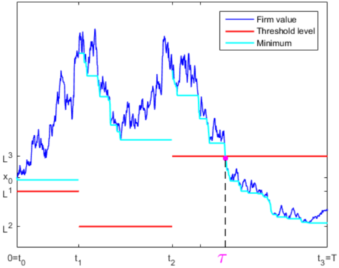
$\mathbf{L} = (L^1, \ldots, L^n)$ are independent of $\mathcal{F}_T$.

Model framework

No Default



Default



Default time $$\tau = \inf\{t \colon X_t \leq L_t\}$$

Running minimum $$M_t = \inf\{X_s \colon s < t\}$$

$$M_{[t,s)} = \inf\{X_u \colon t \leq u < s\}$$

Information structure

Manager's information: $\mathbb{G}^M = (\mathcal{G}_t^M)_{t\in[0,T]}$

Progressive enlargement of $\mathbb{F}$ by the default threshold process L

$$\mathcal{G}_t^M = \mathcal{F}_t \vee \sigma(L_s,\, s \le t),$$

cf. Blanchet-Scalliet, Hillairet & Jiao (2016)

Information structure

Manager's information: $\mathbb{G}^M = (\mathcal{G}_t^M)_{t\in[0,T]}$

Progressive enlargement of $\mathbb{F}$ by the default threshold process L

$$\mathcal{G}_t^M = \mathcal{F}_t \vee \sigma(L_s,\, s \le t),$$

cf. Blanchet-Scalliet, Hillairet & Jiao (2016)

S-Investor's information: $\mathbb{G}^S = (\mathcal{G}_t^S)_{t\in[0,T]}$

The firm's assets are perfectly observed by a **standard** investor:
Progressive enlargement of $\mathbb{F}$ by the random time τ

$$\mathcal{G}_t^S = \mathcal{F}_t \vee \sigma(H_s,\, s \le t),$$

where H is the default indicator process defined by $H_t = \mathbf{1}_{\{t\ge\tau\}}$.

Information structure

D-Investor's information: $\mathbb{G}^D = (\mathcal{G}_t^D)_{t\in[0,T]}$

The firm's assets are only observed at **discrete** time points $0 = T_0 < T_1, \ldots, T_N < T$:

Progressive enlargement of $\mathbb{F}^D$ by the random time τ

$$G_t^D = \mathcal{F}_t^D \vee \sigma(H_s,\, s \le t),$$

where $\mathbb{F}^D = (\mathcal{F}_t^D)_{t\in[0,T]}$ is a sub-filtration of $\mathbb{F}$ given by

$$\mathcal{F}_t^D = \begin{cases} \mathcal{F}_0, & \text{if } t < T_1 \\ \sigma(X_{T_1}, \ldots, X_{T_i}) & \text{if } T_i \le t < T_{i+1} \end{cases}$$

S-Investor's information: pricing defaultable claims

A default-contingent claim is characterized by a pair (C, T), where

- T denotes the maturity date and
- C is a $\mathcal{F}_T$-measurable random variable.

The value of a defaultable claim (C, T) for an S-Investor with progressive information flow $\mathbb{G}^S$ is given by

$$\begin{aligned} V_t &= \mathbb{E}^{\mathbb{P}}\left[\frac{R_t}{R_T}\, C\, \mathbf{1}_{\{\tau > T\}} | \mathcal{G}_t^S\right] \\ &= \frac{\mathbf{1}_{\{\tau > t\}}}{\mathbb{P}(\tau > t|\mathcal{F}_t)} \mathbb{E}^{\mathbb{P}}\left[\frac{R_t}{R_T}\, C\, \mathbf{1}_{\{\tau > T\}} | \mathcal{F}_t\right]. \end{aligned}$$

where R denotes the $\mathbb{F}$-adapted discount factor process.

Example: $C = 1,\ R_t = 1$ (risk-free interest rate $r = 0$)

$$V_t = \mathbb{P}(\tau > T|\mathcal{G}_t^S) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T|\mathcal{F}_t)}{\mathbb{P}(\tau > t|\mathcal{F}_t)}$$

S-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$,

$$\mathbb{P}(\tau > T|\mathcal{G}_t^S) = \mathbf{1}_{\{\tau>t\}} \frac{\mathbb{P}(\tau > T|\mathcal{F}_t)}{\mathbb{P}(\tau > t|\mathcal{F}_t)}$$

For $t \in [t_0, t_1)$ we have

$$\begin{aligned}\mathbb{P}(\tau > T|\mathcal{F}_t) &= \mathbb{P}(L^1 < M_{t_1}, L^2 < M_{[t_1,T)}|\mathcal{F}_t) \\ &= \mathbb{P}(L^1 < M_t, L^1 < M_{[t,t_1)}, L^2 < M_{[t_1,T)}|\mathcal{F}_t)\end{aligned}$$

S-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$,

$$\mathbb{P}(\tau > T | \mathcal{G}_t^S) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T | \mathcal{F}_t)}{\mathbb{P}(\tau > t | \mathcal{F}_t)}$$

For $t \in [t_0, t_1)$ we have

$$\mathbb{P}(\tau > T | \mathcal{F}_t) = \int_0^1 \int_0^\infty \int_0^1 F(\min(M_t, uX_t); wvX_t) f_{M_{T-t_1}}(w) f_{M_{t_1-t}, X_{t_1-t}}(u, v) \, \mathrm{d}w \, \mathrm{d}v \, \mathrm{d}u$$

S-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$,

$$\mathbb{P}(\tau > T|\mathcal{G}_t^S) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T|\mathcal{F}_t)}{\mathbb{P}(\tau > t|\mathcal{F}_t)}$$

For $t \in [t_0, t_1)$ we have

$$\mathbb{P}(\tau > T|\mathcal{F}_t) = \int_0^1 \int_0^\infty \int_0^1 F(\min(M_t, uX_t); wvX_t) f_{M_{T-t_1}}(w) \\ f_{M_{t_1-t}, X_{t_1-t}}(u, v)\, \mathrm{d}w\, \mathrm{d}v\, \mathrm{d}u,$$

where

- $F(\ell_1; \ell_2)$ - joint distribution function of $\mathbf{L} = (L^1, L^2)$

S-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$,

$$\mathbb{P}(\tau > T|\mathcal{G}_t^S) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T|\mathcal{F}_t)}{\mathbb{P}(\tau > t|\mathcal{F}_t)}$$

For $t \in [t_0, t_1)$ we have

$$\mathbb{P}(\tau > T|\mathcal{F}_t) = \int_0^1 \int_0^\infty \int_0^1 F(\min(M_t, uX_t); wvX_t) f_{M_{T-t_1}}(w) \, f_{M_{t_1-t}, X_{t_1-t}}(u, v) \, \mathrm{d}w \, \mathrm{d}v \, \mathrm{d}u,$$

where

- $f_{M_t, X_t}(u, v)$ - joint density function of (M_t, X_t)

$$f_{M_t, X_t}(u, v) = \frac{2v^{m/\sigma^2 - 1} \ln(v/u^2)}{\sigma^3 \sqrt{2\pi} t^{3/2} u} e^{-\frac{m^2 t}{2\sigma^2}} e^{-\frac{\ln^2(v/u^2)}{2\sigma^2 t}},$$

for $u \in (0, 1]$ and $u \leq v$

S-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$,

$$\mathbb{P}(\tau > T | \mathcal{G}_t^S) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T | \mathcal{F}_t)}{\mathbb{P}(\tau > t | \mathcal{F}_t)}$$

For $t \in [t_0, t_1)$ we have

$$\mathbb{P}(\tau > T | \mathcal{F}_t) = \int_0^1 \int_0^\infty \int_0^1 F(\min(M_t, uX_t); wvX_t) f_{M_{T-t_1}}(w) \, f_{M_{t_1-t}, X_{t_1-t}}(u, v) \, \mathrm{d}w \, \mathrm{d}v \, \mathrm{d}u,$$

where

- $f_{M_t}(w)$ - density function of M_t

$$f_{M_t}(w) = \frac{1}{\sigma w \sqrt{t}} \varphi \left(\frac{mt - \frac{\ln w}{\sigma}}{\sqrt{t}} \right) + \frac{e^{2m \frac{\ln w}{\sigma}}}{\sigma w \sqrt{t}} \varphi \left(\frac{mt + \frac{\ln w}{\sigma}}{\sqrt{t}} \right) + \frac{2m}{w\sigma} e^{2m \frac{\ln w}{\sigma}} \Phi \left(\frac{mt + \frac{\ln w}{\sigma}}{\sqrt{t}} \right);$$

S-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$,

$$\mathbb{P}(\tau > T | \mathcal{G}_t^S) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T | \mathcal{F}_t)}{\mathbb{P}(\tau > t | \mathcal{F}_t)}$$

For $t \in [t_0, t_1)$ we have

$$\mathbb{P}(\tau > T | \mathcal{F}_t) = \int_0^1 \int_0^\infty \int_0^1 F(\min(M_t, uX_t); wvX_t) f_{M_{T-t_1}}(w) f_{M_{t_1-t}, X_{t_1-t}}(u, v) \, dw \, dv \, du$$

and

$$\mathbb{P}(\tau > t | \mathcal{F}_t) = \mathbb{P}(L^1 < M_t | \mathcal{F}_t) = F_{L^1}(M_t),$$

where $F_{L^1}(x)$ denotes the distribution function of L^1.

S-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$,

$$\mathbb{P}(\tau > T | \mathcal{G}_t^S) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T | \mathcal{F}_t)}{\mathbb{P}(\tau > t | \mathcal{F}_t)}$$

For $t \in [t_1, T)$ we have

$$\begin{aligned}
\mathbb{P}(\tau > T | \mathcal{F}_t) &= \mathbb{P}(L^1 < M_{t_1}, L^2 < M_{[t_1,T)} | \mathcal{F}_t) \\
&= \mathbb{P}(L^1 < M_{t_1}, L^2 < M_{[t_1,t)}, L^2 < M_{[t,T)} | \mathcal{F}_t) \\
&= \int_0^1 F(M_{t_1}; \min(M_{[t_1,T)}, wX_t)) f_{M_{T-t}}(w) \, dw
\end{aligned}$$

S-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$,

$$\mathbb{P}(\tau > T | \mathcal{G}_t^S) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T | \mathcal{F}_t)}{\mathbb{P}(\tau > t | \mathcal{F}_t)}$$

For $t \in [t_1, T)$ we have

$$\begin{aligned}\mathbb{P}(\tau > T | \mathcal{F}_t) &= \mathbb{P}(L^1 < M_{t_1}, L^2 < M_{[t_1,t)}, L^2 < M_{[t,T)} | \mathcal{F}_t) \\ &= \int_0^1 F(M_{t_1}; \min(M_{[t_1,T)}, wX_t)) f_{M_{T-t}}(w)\, \mathrm{d}w\end{aligned}$$

and

$$\mathbb{P}(\tau > t | \mathcal{F}_t) = \mathbb{P}(L^1 < M_{t_1}, L^2 < M_{[t_1,t)} | \mathcal{F}_t) = F(M_{t_1}; M_{[t_1,t)}).$$

D-Investor's information: pricing defaultable claims

The value of a default-contingent claim (C, T) for an D-Investor with information flow $\mathbb{G}^D$ is given by

$$V_t^D = \mathbb{E}^{\mathbb{P}}\left[\frac{R_t}{R_T}\, C\, \mathbf{1}_{\{\tau>T\}} | \mathcal{G}_t^D\right]$$
$$= \frac{\mathbf{1}_{\{\tau>t\}}}{\mathbb{P}(\tau > t | \mathcal{F}_t^D)} \mathbb{E}^{\mathbb{P}}\left[\frac{R_t}{R_T}\, C\, \mathbf{1}_{\{\tau>T\}} | \mathcal{F}_t^D\right].$$

where R denotes the $\mathbb{F}$-adapted discount factor process.

Example: $C = 1$, $R_t = 1$ (risk-free interest rate $r = 0$)

$$V_t^D = \mathbb{P}(\tau > T | \mathcal{G}_t^D) = \mathbf{1}_{\{\tau>t\}} \frac{\mathbb{P}(\tau > T | \mathcal{F}_t^D)}{\mathbb{P}(\tau > t | \mathcal{F}_t^D)}$$

D-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$, then $\mathbb{P}(\tau > T | \mathcal{G}_t^D) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T | \mathcal{F}_t^D)}{\mathbb{P}(\tau > t | \mathcal{F}_t^D)}$.
For $t \in [t_0, t_1)$, $t \in [T_i, T_{i+1})$ and $t_1 =: T_j \geq T_{i+1}$ we have

$$\begin{aligned} \mathbb{P}(\tau > T | \mathcal{F}_t^D) &= \mathbb{P}(L^1 < M_{t_1}, L^2 < M_{[t_1,T)} | \mathcal{F}_t^D) \\ &= \mathbb{P}(L^1 < M_t, L^1 < M_{[t,t_1)}, L^2 < M_{[t_1,T)} | \mathcal{F}_t^D) \end{aligned}$$

D-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$, then $\mathbb{P}(\tau > T|\mathcal{G}_t^D) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T|\mathcal{F}_t^D)}{\mathbb{P}(\tau > t|\mathcal{F}_t^D)}$.
For $t \in [t_0, t_1)$, $t \in [T_i, T_{i+1})$ and $t_1 =: T_j \geq T_{i+1}$ we have

$$\mathbb{P}(\tau > T|\mathcal{F}_t^D) = \int_0^1 \int_0^\infty \int_0^1 \int_0^{uX_{T_i}} \int_0^{wvX_{T_i}} \mathbb{P}(M_{T_i} > \ell_1|X_{T_1}, ..., X_{T_i}) f(\ell_1; \ell_2) f_{M_{T-t_1}}(w) f_{M_{t_1-T_i}, X_{t_1-T_i}}(u, v) \, d\ell_2 \, d\ell_1 \, dw \, dv \, du$$

D-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$, then $\mathbb{P}(\tau > T|\mathcal{G}_t^D) = \mathbf{1}_{\{\tau>t\}} \frac{\mathbb{P}(\tau>T|\mathcal{F}_t^D)}{\mathbb{P}(\tau>t|\mathcal{F}_t^D)}$.
For $t \in [t_0, t_1)$, $t \in [T_i, T_{i+1})$ and $t_1 =: T_j \geq T_{i+1}$ we have

$$\mathbb{P}(\tau > T|\mathcal{F}_t^D) = \int_0^1 \int_0^\infty \int_0^1 \int_0^{uX_{T_i}} \int_0^{wvX_{T_i}} \mathbb{P}(M_{T_i} > \ell_1 | X_{T_1}, ..., X_{T_i}) f(\ell_1; \ell_2) \\ f_{M_{T-t_1}}(w) f_{M_{t_1-T_i}, X_{t_1-T_i}}(u, v) \, d\ell_2 \, d\ell_1 \, dw \, dv \, du$$

where $f(\ell_1; \ell_2)$ denotes the joint density function of $\mathbf{L} = (L^1, L^2)$;

D-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$, then $\mathbb{P}(\tau > T|\mathcal{G}_t^D) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T|\mathcal{F}_t^D)}{\mathbb{P}(\tau > t|\mathcal{F}_t^D)}$.
For $t \in [t_0, t_1)$, $t \in [T_i, T_{i+1})$ and $t_1 =: T_j \geq T_{i+1}$ we have

$$\mathbb{P}(\tau > T|\mathcal{F}_t^D) = \int_0^1 \int_0^\infty \int_0^1 \int_0^{uX_{T_i}} \int_0^{wvX_{T_i}} \mathbb{P}(M_{T_i} > \ell_1 | X_{T_1}, ..., X_{T_i}) f(\ell_1; \ell_2)$$
$$f_{M_{T-t_1}}(w) f_{M_{t_1 - T_i}, X_{t_1 - T_i}}(u, v)\, d\ell_2\, d\ell_1\, dw\, dv\, du$$

where

$$\mathbb{P}(M_{T_i} > \ell_1 | X_{T_1}, ..., X_{T_i}) = \prod_{j=1}^{i} K_j(\ell_1), \quad \ell_1 < \min\{X_{T_1}, \ldots, X_{T_i}\}$$

and

$$K_j(\ell_1) = \left(1 - \exp\left(\frac{-2}{(T_j - T_{j-1})\sigma^2} \log(\frac{\ell_1}{X_{T_{j-1}}}) \log(\frac{\ell_1}{X_{T_j}})\right)\right)$$

D-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$, then $\mathbb{P}(\tau > T|\mathcal{G}_t^D) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T|\mathcal{F}_t^D)}{\mathbb{P}(\tau > t|\mathcal{F}_t^D)}$.
For $t \in [t_0, t_1)$, $t \in [T_i, T_{i+1})$ and $t_1 =: T_j \geq T_{i+1}$ we have

$$\mathbb{P}(\tau > t|\mathcal{F}_t^D) = \int_0^\infty \Psi\left(\frac{\ell_1}{X_{T_i}}, t - T_i\right) \mathbb{P}(M_{T_i} > \ell_1 | X_{T_1}, ..., X_{T_i}) f_{L_1}(\ell_1)\, d\ell_1,$$

where $f_{L_1}(\ell_1)$ denotes the density function of L^1;

D-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$, then $\mathbb{P}(\tau > T | \mathcal{G}_t^D) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T | \mathcal{F}_t^D)}{\mathbb{P}(\tau > t | \mathcal{F}_t^D)}$.
For $t \in [t_0, t_1)$, $t \in [T_i, T_{i+1})$ and $t_1 =: T_j \geq T_{i+1}$ we have

$$\mathbb{P}(\tau > t | \mathcal{F}_t^D) = \int_0^\infty \Psi\left(\frac{\ell_1}{X_{T_i}}, t - T_i\right) \mathbb{P}(M_{T_i} > \ell_1 | X_{T_1}, ..., X_{T_i}) \\ f_{L_1}(\ell_1)\, d\ell_1,$$

where for $z < 1$ and $t > 0$

$$\begin{aligned} \Psi(z, t) &= \mathbb{P}(M_t > z) \\ &= \Phi\left(\frac{mt - \frac{\log(z)}{\sigma}}{\sqrt{t}}\right) - e^{\frac{2m\log(z)}{\sigma}} \Phi\left(\frac{mt + \frac{\log(z)}{\sigma}}{\sqrt{t}}\right), \end{aligned}$$

$\Psi(z, t) = 0$ for $t \geq 0$, $z \geq 1$ and $\Psi(z, 0) = 1$ for $z < 1$.

D-Investor's information: survival probability

Let $n = 2$ and $x_0 = 1$, then $\mathbb{P}(\tau > T|\mathcal{G}_t^D) = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{P}(\tau > T|\mathcal{F}_t^D)}{\mathbb{P}(\tau > t|\mathcal{F}_t^D)}$.

For $t \in [t_1, T)$, $t \in [T_i, T_{i+1})$ and $t_1 =: T_j \geq T_i$ we have

$$\mathbb{P}(\tau > T|\mathcal{F}_t^D) = \int_0^\infty \int_0^\infty \mathbb{P}(M_{T_j} > \ell_1, M_{[T_j, T_i)} > \ell_2 | X_{T_1}, \ldots, X_{T_i}) \Psi\left(\frac{\ell_2}{X_{T_i}}, T - T_i\right) f(\ell_1; \ell_2)\, \mathrm{d}\ell_2\, \mathrm{d}\ell_1,$$

where for $\ell_1 < \min\{X_{T_1}, \ldots, X_{T_j}\}$ and $\ell_2 < \min\{X_{T_j}, \ldots, X_{T_i}\}$

$$\mathbb{P}(M_{T_j} > \ell_1, M_{[T_j, T_i)} > \ell_2 | X_{T_1}, \ldots, X_{T_i}) = K_1(\ell_1) \cdot \ldots \cdot K_j(\ell_1) \cdot K_{j+1}(\ell_2) \cdot \ldots \cdot K_i(\ell_2),$$

where

$$K_i(\ell) = \left(1 - \exp\left(\frac{-2}{(T_i - T_{i-1})\sigma^2} \log\left(\frac{\ell}{X_{T_{i-1}}}\right) \log\left(\frac{\ell}{X_{T_i}}\right)\right)\right).$$

Numerical results

Parameters

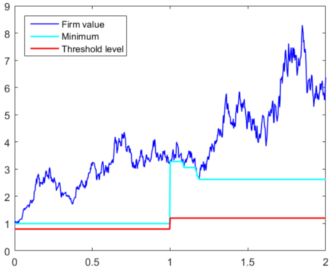
- Two time periods: $0 = t_0 < t_1 = 1 < t_2 = 2 = T$;
- Asset process: $\mu = 0.05$, $\sigma = 0.8$ and $x_0 = 1$;
- Default barrier: L^1 and L^2 are independent exponentially distributed with $\lambda_1 = 1.5$ and $\lambda_2 = 1$.

Value process of a defaultable bond with zero recovery

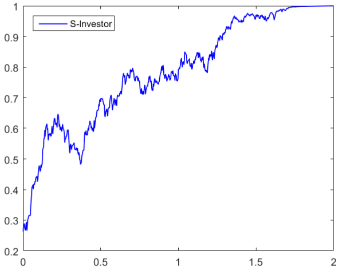
Discount factor process: $R_t = e^{rt}$, $r = 0.01$

$$V_t = e^{-r(T-t)}\,\mathbb{P}(\tau > T|\mathcal{G}_t^S)$$
$$= e^{-r(T-t)}\,\mathbf{1}_{\{\tau>t\}}\frac{\mathbb{P}(\tau > T|\mathcal{F}_t)}{\mathbb{P}(\tau > t|\mathcal{F}_t)}$$

Figure: No default during [0,2], $L^1 = 0.8$, $L^2 = 1.2$



(a) Firm value



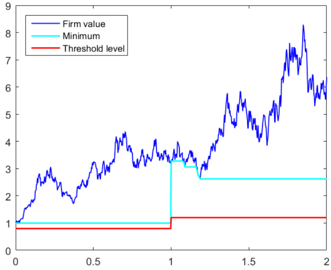
(b) Bond price

Value process of a defaultable bond with zero recovery

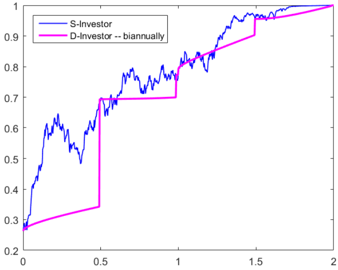
Discount factor process: $R_t = e^{rt}$, $r = 0.01$

$$V_t^D = e^{-r(T-t)}\,\mathbb{P}(\tau > T|\mathcal{G}_t^D)$$
$$= e^{-r(T-t)}\,\mathbf{1}_{\{\tau>t\}}\frac{\mathbb{P}(\tau > T|\mathcal{F}_t^D)}{\mathbb{P}(\tau > t|\mathcal{F}_t^D)}$$

Figure: No default during [0,2], $L^1 = 0.8$, $L^2 = 1.2$



(a) Firm value



(b) Bond price

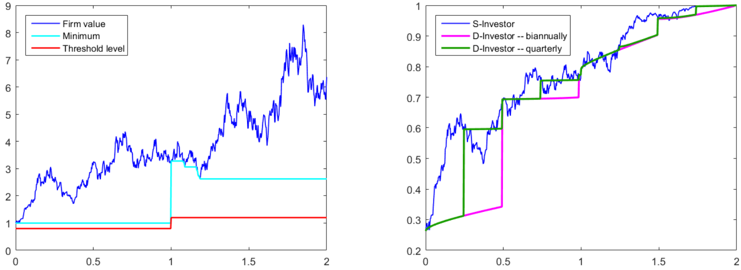
Value process of a defaultable bond with zero recovery

Discount factor process: $R_t = e^{rt}$, $r = 0.01$

$$V_t^D = e^{-r(T-t)}\,\mathbb{P}(\tau > T|\mathcal{G}_t^D)$$

$$= e^{-r(T-t)}\,\mathbf{1}_{\{\tau>t\}}\frac{\mathbb{P}(\tau > T|\mathcal{F}_t^D)}{\mathbb{P}(\tau > t|\mathcal{F}_t^D)}$$

Figure: No default during [0,2], $L^1 = 0.8$, $L^2 = 1.2$



(a) Firm value

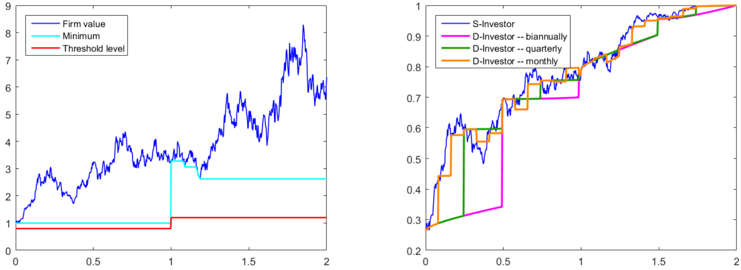
(b) Bond price

Value process of a defaultable bond with zero recovery

Discount factor process: $R_t = e^{rt}$, $r = 0.01$

$$V_t^D = e^{-r(T-t)}\,\mathbb{P}(\tau > T|\mathcal{G}_t^D)$$
$$= e^{-r(T-t)}\,\mathbf{1}_{\{\tau>t\}}\frac{\mathbb{P}(\tau > T|\mathcal{F}_t^D)}{\mathbb{P}(\tau > t|\mathcal{F}_t^D)}$$

Figure: No default during [0,2], $L^1 = 0.8$, $L^2 = 1.2$



(a) Firm value

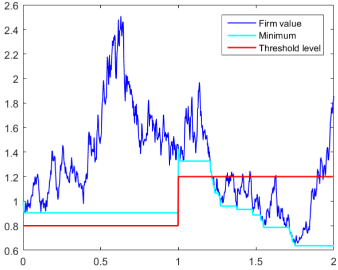
(b) Bond price

Value process of a defaultable bond with zero recovery

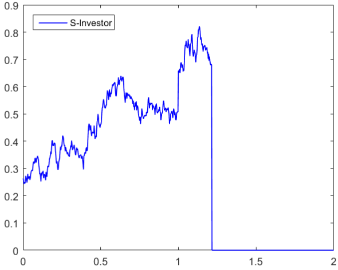
Discount factor process: $R_t = e^{rt}$, $r = 0.01$

$$V_t = e^{-r(T-t)}\,\mathbb{P}(\tau > T|\mathcal{G}_t^S)$$
$$= e^{-r(T-t)}\,\mathbf{1}_{\{\tau>t\}}\frac{\mathbb{P}(\tau > T|\mathcal{F}_t)}{\mathbb{P}(\tau > t|\mathcal{F}_t)}$$

Figure: Default during [1,2], $L^1 = 0.8$, $L^2 = 1.2$



(a) Firm value



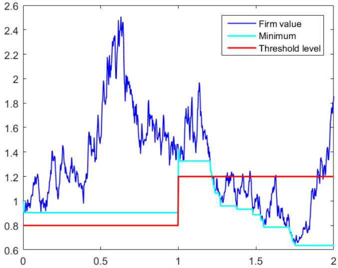
(b) Bond price

Value process of a defaultable bond with zero recovery

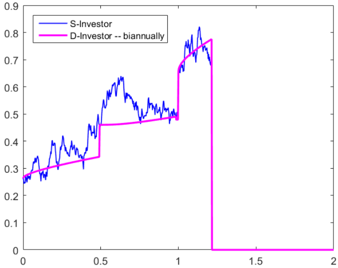
Discount factor process: $R_t = e^{rt}$, $r = 0.01$

$$V_t^D = e^{-r(T-t)}\,\mathbb{P}(\tau > T|\mathcal{G}_t^D)$$
$$= e^{-r(T-t)}\,\mathbf{1}_{\{\tau>t\}}\frac{\mathbb{P}(\tau > T|\mathcal{F}_t^D)}{\mathbb{P}(\tau > t|\mathcal{F}_t^D)}$$

Figure: Default during [1,2], $L^1 = 0.8$, $L^2 = 1.2$



(a) Firm value



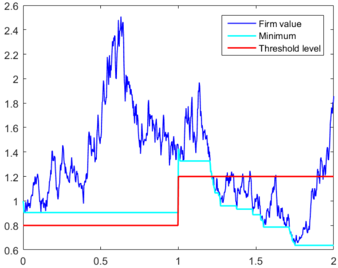
(b) Bond price

Value process of a defaultable bond with zero recovery

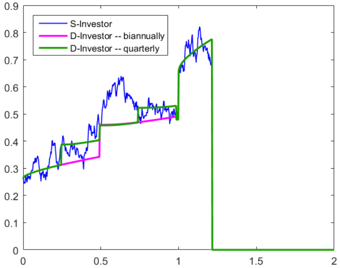
Discount factor process: $R_t = e^{rt}$, $r = 0.01$

$$V_t^{D} = e^{-r(T-t)}\,\mathbb{P}(\tau > T|\mathcal{G}_t^{D})$$
$$= e^{-r(T-t)}\,\mathbf{1}_{\{\tau>t\}}\frac{\mathbb{P}(\tau > T|\mathcal{F}_t^{D})}{\mathbb{P}(\tau > t|\mathcal{F}_t^{D})}$$

Figure: Default during [1,2], $L^1 = 0.8$, $L^2 = 1.2$



(a) Firm value



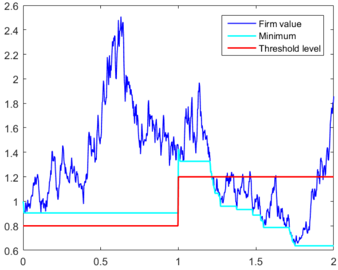
(b) Bond price

Value process of a defaultable bond with zero recovery

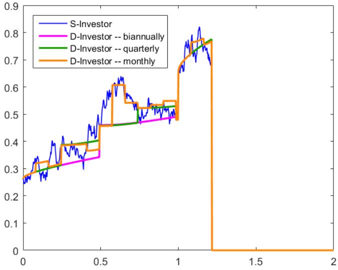
Discount factor process: $R_t = e^{rt}$, $r = 0.01$

$$V_t^D = e^{-r(T-t)}\,\mathbb{P}(\tau > T|\mathcal{G}_t^D)$$
$$= e^{-r(T-t)}\,\mathbf{1}_{\{\tau>t\}}\frac{\mathbb{P}(\tau > T|\mathcal{F}_t^D)}{\mathbb{P}(\tau > t|\mathcal{F}_t^D)}$$

Figure: Default during [1,2], $L^1 = 0.8$, $L^2 = 1.2$



(a) Firm value



(b) Bond price

References

- Bielecki, T.R., Rutkowski, M. (2002). Credit Risk: Modeling, Valuation and Hedging, Springer.
- Blanchet-Scalliet, C., Hillairet, C., Jiao, Y. (2016). Successive Enlargement of Filtrations and Application to Insider Information, preprint.
- Duffie, D., Lando, D. (2001). Term Structures of Credit Spreads with Incomplete Accounting Information, Econometrica, 69, 633-664.
- Giesecke K, Goldberg L. R. (2008). The Market Price of Credit Risk : The Impact of Asymmetric Information.
- Hillairet, C., Jiao, Y. (2012). Credit Risk with Asymmetric Information on the Default Threshold, STOCHASTICS, 84(2-3), 135.
- Jeanblanc, M., Valchev, S. (2005). Partial Information and Hazard Process, International Journal of Theoretical and Applied Finance, 8(6), 807–838.
- Lakner, P., Liang, W. (2008). Optimal Investment in a Defaultable Bond, Mathematics and Financial Economics, 1(3/4), 283-310.